Bicyclist Problem

This is a more thorough way to do this problem. Create a function f(t) that represents the difference in the two bicyclists distance as they go from 0 to 2hr 15 min. Since they start together and end together, their distances would be the same at the endpoints, so f(0)=0 and f(2.25)=0. We can assume that f is continuous and differentiable. Since f(0) = f(2.25) then Rolle’s Thm is satisfied and there is some point, c between 0 and 2.25 for which f’(c)=0. This means that the difference in their velocities is zero at c, which proves that there is some point where their velocities are the same.

 show that f has exactly one real zero.

IVT : Since f(1)=4 and f(-1)=-2 we know that there is at least one zero, c between -1 and 1 by the IVT.

Suppose f(x) has more than one zero. Suppose it has another zero, d. Then f(c)=f(d)=0. Since this meets the condition of Rolle’s Thm, we know that there is a point m, in (c,d) such that f’(m)=0,

But  is always positive and never zero. Since this is a contradiction, then we know that the assumption that f(x) has more than one zero is false. Therefore f has exactly one real zero